

# ANISOTROPIC SHEAR STRESS $\sigma_{xy}$ EFFECTS IN THE BASAL PLANE OF Sr<sub>2</sub>RuO<sub>4</sub>

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## ABSTRACT

In the present paper following the previous work (Walker and Contreras, 2002; Walker, 1980; Walker *et al.*, 2001; Contreras, 2006) we calculate the jumps for the thermal expansion  $\alpha_{axy}$ , the specific heat  $C_{axy}$ , and the elastic compliance  $S_{xyxy}^{\sigma_{xy}}$  in the basal plane of Sr<sub>2</sub>RuO<sub>4</sub>. We use here the 4<sup>th</sup> rank tensor notation because of the Voigt notation, where the stress and strain are treated differently. Henceforth, we clarify some issues regarding a Ginzburg-Landau analysis suitable to explain the sound speed experiments (Lupien, 2002), and partially the strain experiments (Hicks *et al.*, 2014; Steppke *et al.*, 2017) in strontium ruthenate. We continue to propose the following: (1) the discontinuity in the elastic constant  $C_{xyxy}$  of the tetragonal crystal Sr<sub>2</sub>RuO<sub>4</sub> gives an unambiguous experimental evidence that the Sr<sub>2</sub>RuO<sub>4</sub> superconducting order parameter  $\Psi$  has two components with a broken time-reversal symmetry state, and (2) the  $\gamma$  band couples the anisotropic electron-phonon interaction to the [xy] in-plane shear stress in Sr<sub>2</sub>RuO<sub>4</sub> (Walker *et al.*, 2001; Contreras, 2006).

PACS numbers: 74.20.De; 74.70.Rp, 74.70.Pq

Keywords: Shear stress, time-reversal symmetry, ehrenfest relations, elastic constants, thermal expansion.

# INTRODUCTION

In Sr<sub>2</sub>RuO<sub>4</sub>, the electrons in the Cooper pairs are bound in spin triplets, where the spins are lying on the basal plane and the pair orbital momentum is directed along the zdirection. Henceforth, the order parameter  $\Psi$  is represented by a vector **d**(**k**) (of the type  $k_x \pm ik_y$ ) (Maeno *et al.*, 1994; Maeno *et al.*, 2001; MacKenzie and Maeno, 2003; Rice and Sigrist, 1995).

Based on the results of the Knight-shift experiment performed through T<sub>c</sub> (Ishida et al., 1998; Duffy et al., 2000), it has been proposed that Sr<sub>2</sub>RuO<sub>4</sub> is a triplet superconductor, also it has been reported by Luke et al. (1998) that  $\Psi$  breaks time-reversal symmetry, which constitutes another key feature of unconventionality. The  $Sr_2RuO_4$  elastic constants  $C_{xyxy}$  have been carefully measured as the temperature T is lowered through  $T_c$ , showing the existence of small step in the transverse sound mode T[100] (Lupien, 2002). This experimental result theoretically implies that  $\Psi$  has two different components with a time-reversal symmetry broken state (Walker and Contreras, 2002). Similar conclusions from a muon-spin relaxation ( $\mu$ SR) experiment were reported by Luke et al. (1998). Recently, experiments on the effects of uniaxial strain  $\epsilon_{xy}$ , were performed by Hicks *et al.* (2014),

reporting that for  $Sr_2RuO_4$  the symmetry-breaking field can be controlled experimentally.

Additionally, a most recently experiment (Steppke et al., 2017) found that the transition temperature  $T_c$  in the superconductor Sr<sub>2</sub>RuO<sub>4</sub> rises dramatically under the application of a planar anisotropic strain, followed by a sudden drop beyond a larger strain. Furthermore, recent theoretical work suggests that those recent experiments tuned the Fermi surface topology efficiently by applying planar anisotropic strain emphasizing again, the point of view that in-plane effects (even by means of a more complicated renormalization group theory framework) also shows clear evidence of a symmetry broken stated in Sr<sub>2</sub>RuO<sub>4</sub>. Furthermore, they reported a rapid initial increase in the superconducting transition temperature  $T_c$ , that the authors associated with the evolution of the Fermi surface toward a Lifshitz-Fermi surface reconstruction under an increasing strain (Liu et al., 2017).

Here, we aim to clarify some particular concepts and methods following an elastic phenomenological (GL) approach (Walker, 1980; Landau and Lifshitz, 1980; Landau and Lifshitz, 1970; Testardi, 1971; Auld, 1990; Musgrave, 1970; Philip, 1987). First, it is natural to point out the differences between using stress or strain (which is the response of a system to applied stress; also, according

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to Dr. Grechka explanation, fiber optics yields dynamic fidelity of a fraction of a nanostrain, and tiltmeters yield comparable static fidelity) to explain a time-reversal symmetry broken state. Second, there is a claim (Liu et al., 2017) that only the  $\gamma$  band responses to the strain sensitively, and we emphasize that this physical phenomenon is caused by the  $\gamma$  band coupling of the anisotropic electron-phonon interaction to the [xy] plane (Walker et al., 2001; Contreras, 2006). Third, we do not expand our analysis to a Lifshitz reconstruction of the Fermi surface mainly because we do not have experimental evidences that show a topological Lifshitz transition in Sr<sub>2</sub>RuO<sub>4</sub> even in its normal state, neither we have observed a generalized topological transition in Sr<sub>2</sub>RuO<sub>4</sub>. In our opinion, a two-dimensional Fermi contour evolution under an applied external strain as the one for the  $\gamma$  band in Sr<sub>2</sub>RuO<sub>4</sub> needs further interpretations in terms of a topological electronic Lifshitz phase transition (Lifshitz, 1960; Kaganov and Lifshitz, 1979; Kaganov and Contreras, 1994; Kaganov and Nurmagambetov, 1982). We remember that  $\sigma_{ik} = -p\delta_{ik}$ shows how pressure and stress are in general related, the stress  $\sigma_{ik}$  becomes the delta function  $\delta_{ik}$  if a volumetric pressure is applied to a sample (Landau and Lifshitz, 2009).

These experimental results and theoretical interpretations need to clarify moderately in order to unify several theoretical criteria which try to explain the changes occurring in the  $C_{xyxy}$  elastic constant at  $T_c$  (Lupien, 2002; Hicks *et al.*, 2014; Steppke *et al.*, 2017).



Fig. 1. The phase diagram sketch showing the upper  $T_{c+}$ , lower  $T_{c-}$ , and zero  $T_{c0}$  superconducting transition temperatures as a function of the shear stress  $-\sigma_{xy}(T)$ . Notice that at  $T_{c0}$  the derivative  $\frac{dT_{c0}}{d\sigma_{xy}}$  does not exist.

Thus, the aim of this short note is to clarify again that an elastic Ginzburg-Landau phenomenological approach partially demonstrates that  $Sr_2RuO_4$  is an unconventional superconductor with a two-component order parameter  $\Psi$ 

(Walker and Contreras, 2002; Contreras, 2006). We based our interpretation on a  $Sr_2RuO_4$  (T[100]) transversal response-impulse mode experimentally measured as the temperature T is lowered through  $T_c$  showing only an small step change (Lupien, 2002) (for that particular result, please, see the bottom panel in (Lupien, 2002), namely in Figures 5.7 and 5.8 on pages 138 and 139.) The result clearly shows a discontinuity in the T[100] mode.

Here, let us mention that a different theory of  $Sr_2RuO_4$ elastic properties was presented in (Sigrist, 2002). However, the approach followed does not take into account the splitting of  $T_c$  due to the shear  $\sigma_{xy}$ , and directly calculates the jumps at zero stress, where the derivative of T with respect to  $\sigma_{xy}$  does not exist (Walker, 1980), see in Figure 1.

## Shear stress $\sigma_{xy}$ analysis

In this section, we make use of the  $4^{th}$  rank tensor notation because the Voigt notation has a disadvantage; the stress and strain are treated differently. Voigt mapping only preserves the elastic stiffnesses. We also call the uniaxial shear stress as shear stress only because the effect observed is in the basal plane [xy]. As was stated previously (Walker and Contreras, 2002; Contreras, 2006) when shear stress  $\sigma_{xy}$  is applied to the basal plane of Sr<sub>2</sub>RuO<sub>4</sub>, the crystal tetragonal symmetry is broken, and a second-order transition to a superconducting state occurs. Accordingly, for this case the analysis of the sound speed behavior at  $T_c$  also requires a systematic study of the two successive second order phase transitions, see in figure 2. Hence, the  $C_{xyxy}$  discontinuity (Lupien, 2002) at  $T_c$ , can be explained in this context. Due to the absence of discontinuity in  $S_{xyxy}$  for any of the one-dimensional  $\Gamma$ representations, the superconductivity in Sr<sub>2</sub>RuO<sub>4</sub> must be described by the two-dimensional irreducible representation  $E_{2u}$  of the tetragonal point group  $D_{4h}$ (Walker and Contreras, 2002).



Fig. 2. The sketch showing the line of transition temperatures  $T_c$  along of a second order phase transition. The state of the strain  $e_{xy}$  shown in the figure, the entropy

 $S_0$ , and the volume V are continuous functions along the line of the second order phase transition (Walker, 1980; Contreras, 2006).

If there is a double transition, the derivative of  $T_c$  with respect to  $\sigma_{xy}$  (i.e.  $dT_c/d\sigma_{xy}$ ) is different for each of the two transition lines (see the  $T_c$ - $\sigma_{xy}$  phase diagram in Figure 2.) At each of these transitions, the specific heat  $C_{\alpha xy}$ , the thermal expansion  $\alpha_{\alpha xy}$ , and the elastic compliance  $S_{xyxy}^{\sigma_{xy}}$ show discontinuities (Walker and Contreras, 2002), the sum of them gives the correct expressions for the discontinuities at zero shear stress, where the Ehrenfest relations do not hold.

In the case of an applied shear stress  $\sigma_{xy}$ , the change for the in-plane Gibbs free energy is given by

$$\begin{aligned} \Delta G_{\sigma_{xy}} &= \alpha (|\psi_x|^2 + |\psi_y|^2) + \sigma_{xy} d_{xyxy} (\psi_x \psi_y^* + \\ \psi_x^* \psi_y) + \frac{b_1}{4} (|\psi_x|^2 + |\psi_y|^2)^2 + b_2 |\psi_x|^2 |\psi_y|^2 + \\ \frac{b_3}{2} (\psi_x^2 \psi_y^{*2} + \psi_x^{*2} \psi_y^2) \\ (1) \end{aligned}$$

where the  $d_{xyxy}$  term couples the stress  $\sigma_{xy}$  to the order parameter, the thermal expansion coefficient  $\alpha = \alpha'(T - T_{c0})$ , and the minimization of  $\Delta G_{\sigma xy}$  is performed by substituting the general expression for  $\Psi$  as was previously calculated (Contreras, 2006; Sigrist, 2002). Therefore,  $\Delta G_{\sigma xy}$  becomes

$$\Delta G_{\sigma_{xy}} = \alpha \left( \eta_x^2 + \eta_y^2 \right) + 2\eta_x \eta_y \sigma_{xy} \sin(\phi) \, d_{xyxy} + \frac{b_1}{4} (\eta_x^2 + \eta_y^2)^2 + (b_2 - b_3) \eta_x^2 \eta_y^2 + 2b_3 \eta_x^2 \eta_y^2 \sin^2(\phi)$$
(2)

In the presence of  $\sigma_{xy}$ , the second order term determines the phase below  $T_{c+}$ , which is characterized by  $\psi_x$  and by  $\psi_y = 0$ . As the temperature is lowered below  $T_{c-}$ , depending of the value of  $b_3$  a second component  $\psi_y$  may appear. If at  $T_{c-}$  a second component occurs, the fourth order terms in equation (2) will dominate. Thus, for very low T's, or for  $\sigma_{xy} \rightarrow 0$ , a time-reversal symmetrybreaking superconducting state may emerge. The analysis of equation (2) depends on the relation between the coefficients  $b_2$  and  $b_3$ . It also depends on the values of the quantities  $\eta_x$  and  $\eta_y$ , and of the phase  $\phi$ . If  $b_3 \le 0$ , and  $\eta_x$ and  $\eta_v$  are both nonzero, the state with minimum energy has a phase  $\phi = \pi/2$ . The transition temperature is obtained from equation (2), by performing the following canonical transformations:  $\eta_x = (\eta_\mu + \eta_\xi)/2^{1/2}$  and  $\eta_y = (\eta_\mu - \eta_\xi)/2^{1/2}$ . After their substitution, equation (2) becomes

$$\begin{aligned} \Delta G_{\sigma_{xy}} &= \alpha_{+}\eta_{\xi}^{2} + \alpha_{-}\eta_{\mu}^{2} + \frac{1}{4}(\eta_{\xi}^{2} + \eta_{\mu}^{2})^{2} + (b_{2} + b_{3})(\eta_{\xi}^{2} - \eta_{\mu}^{2})^{2} \end{aligned}$$
(3)

As it was before done,  $\eta_{\xi} = \eta \sin(\chi)$  and  $\eta_{\mu} = \eta \cos(\chi)$ , equation (3) takes the following form:

$$\Delta G_{\sigma_{xy}} = \alpha_{+} \eta^{2} \sin^{2}(\chi) + \alpha_{-} \eta^{2} \cos^{2}(\chi) + \frac{\eta^{*}}{4} [b_{1} + (b_{2} + b_{3}) \cos^{2}(2\chi)]$$
(4)

 $\Delta G_{\alpha xy}$  is minimized if  $\cos(2\chi) = 1$ , this is, if  $\chi = 0$ . Also, in order for the phase transition to be of second order, b', defined as  $b' \equiv b_1 + b_2 + b_3$ , must be larger than zero. Therefore, if  $\sigma_{xy}$  is nonzero, the state with the lowest free energy corresponds to  $b_3 < 0$ , the phase  $\phi$  equals to  $\pi/2$ , and  $\Psi$  of the form:

$$(\psi_x, \psi_y) \approx \eta(exp(i\phi/2), exp(-i\phi/2))$$
(5)

In phase 1 in Figure 2 there is  $\phi = 0$  and *T* is lowered below  $T_{c-}$ . In phase 2,  $\phi$  grows from 0 to approximately  $\pi/2$ . The two transition temperatures  $T_{c+}$  and  $T_{c-}$  are obtained to be:

$$T_{c+}(\sigma_{xy}) = T_{c0} - \frac{\sigma_{xy}}{\alpha'} d_{xyxy}$$
(6)
$$T_{c}(\sigma_{c}) = T_{c0} + \frac{b\sigma_{xy}}{\alpha'} d_{xyxy}$$

$$I_{c-}(\sigma_{xy}) = I_{c0} + \frac{1}{2b_3 \alpha'} a_{xyxy}$$
(7)

The derivative of  $T_{c^+}$  with respect to  $\sigma_{xy}$ , and the discontinuity in  $C^+_{\sigma_{xy}}$  at  $T_{c^+}$  are, respectively:

$$\frac{dT_{c+}}{d\sigma_{xy}} = -\frac{d_{xyxy}}{\alpha'}$$
(8)

$$\Delta C^+_{\sigma_{\chi y}} = -\frac{2\alpha'^2 T_{C^+}}{b'} (9)$$

After applying the Ehrenfest relations (Landau and Lifshitz, 1980), the results for  $\Delta \alpha_{\alpha xy}$  and  $\Delta S_{xyxy}$  at  $T_{c+}$  are:

$$\Delta \alpha_{\sigma_{XY}}^{+} = -\frac{2\alpha' d_{XYXY}}{b'}$$
(10)

$$\Delta S_{xyxy}^{+} = -\frac{2d_{xyxy}^{2}}{b'}$$
(11)

For  $T_{c-}$ , the derivative of this transition temperature with respect to  $\sigma_{xy}$ , and the discontinuities in the specific heat, thermal expansion and elastic stiffness respectively are:

$$\frac{dT_{c-}}{d\sigma_{xy}} = \frac{bd_{xyxy}}{2b_3\alpha'} \quad (12)$$
$$\Delta C^-_{\sigma_{xy}} = -\frac{4\alpha'^2 b_3 T_{c-}}{bb'} (13)$$

$$\Delta \alpha_{\sigma_{xy}}^{-} = \frac{2\alpha' d_{xyxy}}{b'} (14)$$
$$\Delta S_{xyxy}^{-} = -\frac{b d_{xyxy}^{2}}{b' b_{3}} (15)$$

For the case of  $\sigma_{xy}$ , because the derivative of  $T_c$  with respect to  $\sigma_{xy}$  is not defined at zero stress point (see in Figure 1), the Ehrenfest relations do not hold at  $T_{c0}$ . Thus, the discontinuities occurring at  $T_{c0}$ , in the absence of  $\sigma_{xy}$ , are calculated by adding the expressions obtained for the discontinuities at  $T_{c+}$  and  $T_{c-}$ ,

$$\Delta C_{\sigma_{xy}}^{0} = -\frac{2T_{c0}\alpha'^{2}}{b} (16)$$
$$\Delta S_{xyxy}^{0} = -\frac{d_{xyxy}^{2}}{b_{3}} (17)$$
$$\Delta \alpha_{\sigma_{xy}}^{0} = 0 \quad (18)$$

In this case there is no discontinuity for the thermal expansion of  $Sr_2RuO_4$  at zero stress  $\alpha_{\sigma_{xy}}^0$  that is another physical feature we previously predicted in (Walker and Contreras, 2002), see in Figure 3. Some experimental studies on the changes in the thermal expansion coefficient  $\alpha_i$  below  $T_c$  in the HTS reported in (Asahi *et al.*, 1997) that an additive lattice jump was found to appear spontaneously at  $T_c$  for a high  $T_c$  compound with one-component order parameter.



Fig. 3. The schematic dependence of the thermal expansion on the temperature for  $Sr_2RuO_4$ . Notice the two jumps in the in plane thermal expansion coefficient near the transition temperatures  $T_{c+}$  and  $T_{c-}$ . The sketch shows two jumps of the same magnitude but they have opposite signs, and their sum cancels out at  $T_{c0}$ . This happens for a two-component order parameter  $\Psi$ .

Since the phase diagram was determined as a function of  $\sigma_{xy}$ , rather than as a function of the shear strain  $\epsilon_{xy}$ , see in Figure 1, in this work, as in (Walker and Contreras, 2002; Contreras, 2006), we make use of the 6 × 6 elastic compliance matrix S, and also of the full range tensor

notation. However, the sound speed measurements (Lupien, 2002; Lupien *et al.*, 2001) are best interpreted in terms of the elastic stiffness tensor C with the matrix elements  $C_{ijkl}$ , which is the inverse of the elastic compliance matrix S (Nye, 1989). However, the strain is easier to measure than the stress because fiber optics yields dynamic fidelity of a fraction of a nanostrain. Here we have to mention the explanation by Dr. Grechka: fiber optics yields dynamic fidelity of a fraction of a nanostrain, and tiltmeters yield comparable static fidelity.

Therefore, it is important to be able to obtain the discontinuities in the elastic stiffness matrix in terms of the elastic compliance matrix for the shear stress case. Thus, close to the transition line we follow (Walker, 1980):  $C(T_c + 0^+) = C(T_c - 0^+) + \Delta C$  and  $S(T_c + 0^+) = S(T_c - 0^+) + \Delta S$ , where  $0^+$  is positive and infinitesimal. By making use of the fact that  $C(T_c + 0^+) S(T_c + 0^+) = \hat{1}$ , where  $\hat{1}$  is the unit matrix, it is shown that, to first order, the discontinuities satisfy,  $\Delta C \approx - C \Delta S C$ . In this manner, it is found that, for instance, at  $T_{c^+}$ , the expressions that define the jumps for the discontinuities in elastic stiffness and compliances, due to an external stress, have either a positive or a negative value. In this way,  $\Delta S_{xyxy}$  must have a negative sign; while  $\Delta C_{xyxy}$  must have a positive sign.

#### **Conclusive remarks**

The most noteworthy outcome of this short note is that the observation of a discontinuity in the elastic constant  $C_{xyxy}$ (Lupien, 2002; Lupien et al., 2001) is an evidence that the order parameter  $\Psi$  in Sr<sub>2</sub>RuO<sub>4</sub> has two components as the theoretical GL analysis predicts. Also, the theoretical indicator that the sum of the jumps  $\Delta \alpha^+_{\sigma_{XY}} + \Delta \alpha^-_{\sigma_{XY}} = 0$ for the in-plane thermal expansion coefficient (Walker and Contreras, 2002), see in Figure 3. Hence, the use of Sr<sub>2</sub>RuO<sub>4</sub> as a material in detailed studies of superconductivity symmetry-breaking effects has significant advantages because is described by a twocomponent order parameter. Nevertheless, determining from Sr<sub>2</sub>RuO<sub>4</sub> experimental measurements the magnitude of the parameters in the Ginzburg-Landau model is complicated (Walker and Contreras, 2002).

In the experimental work of the sound velocity measurements (Lupien, 2002; Lupien *et al.*, 2001) in  $Sr_2RuO_4$  a discontinuity in the behavior for  $C_{xyxy}$  below  $T_{c0}$ , without a significant change in the sound speed slope as *T* goes below 1 Kelvin was understood as a signature of an unconventional transition to a superconducting phase (Walker and Contreras, 2002; Lupien, 2002; Sigrist, 2002). Thus, this set of previous results, and other recent results (Hicks *et al.*, 2014; Steppke *et al.*, 2017; Liu *et al.*, 2017; Acharya *et al.*, 2018), considers  $Sr_2RuO_4$  as a strong candidate for a detailed experimental investigation of the effects of a symmetry-breaking field by means of

strain or stress experimental measurements. We also suggest that an in-plane thermal expansion measurement at zero uniaxial shear stress might further clarify any previous interpretation.

## ACKNOWLEDGMENTS

P. Contreras wishes to express his gratitude to Professors C. Lupien and L. Taillefer at the Universite de Sherbrooke from for stimulating discussions regarding their experimental results several years ago at the University of Toronto. We also acknowledge Dr. Vladimir Grechka from Marathon Oil for clarifying certain aspects regarding the use of the shear strain for experimental purposes.

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#### Received: Jan 21, 2019; Revised: April 5, 2019; Accepted: May 8, 2019

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